Indian Statistical Institute, Bangalore Centre
B.Math. (I Year) : 2011-2012
Semester I : Mid-Semestral Examination
Probability Theory I

21.9.2011 Time: $2\frac{1}{2}$ hours. Maximum Marks : 80

Note: The paper carries 83 marks. Any score above 80 will be taken as 80. State clearly the results you are using in your answers.

1. [10+10=20 marks] Two persons, A and B, shoot at a target independently. Suppose A hits the target with probability 0.4, and B hits it with probability 0.7.

(i) Given that the target is hit, find the probability that A hit it.
(ii) Given that exactly one person hit the target, find the probability that A hit it.

2. [9+7+6=22 marks] From a standard pack of 52 cards, 3 cards are drawn at random without replacement. Let X denote the number of aces, and Y the number of kings drawn.

(i) Find the joint discrete density function of X, Y.
(ii) Find the marginal discrete density functions.
(iii) Are X and Y independent? Why?

3. [13 marks] Let X and Y be independent discrete random variables taking values in $\mathbb{Z}$. Show that X^4 and Y^2 are independent random variables.

4. [10+10+8=28 marks] (i) Let $x_n, n = 1, 2, 3, \ldots$ be a sequence of real numbers such that $x_{n+1} \leq x_n$ for all n, and $\lim_{n\to\infty} x_n = x \in \mathbb{R}$. Let F be the distribution function of a real valued discrete random variable. Show that $\lim_{n\to\infty} F(x_n) = F(x)$.
(ii) If the sequence $\{x_n\}$ in (i) above is such that $x_{n+1} \geq x_n$ for all n, does the sequence $\{F(x_n)\}$ converge? Is the conclusion in (i) correct?
(iii) Draw the graph of the distribution function of the binomial distribution with parameters $n = 4, p = \frac{1}{2}$.